# GPS-Like Phasing Control of the Space Solar Power System Transmission Array

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**Abstract**

The problem of phasing of the Space Solar Power System's transmission array has been addressed by developing a GPS-like radio navigation system. The goal of this system is to provide power transmission phasing control for each node of the array that causes the power signals to add constructively at the ground reception station. The phasing control system operates in a distributed manner, which makes it practical to implement. A leader node and two radio navigation beacons are used to control the power transmission phasing of multiple follower nodes. The necessary one-way communications to the follower nodes are implemented using the RF beacon signals. The phasing control system uses differential carrier phase relative navigation/timing techniques. A special feature of the system is an integer ambiguity resolution procedure that periodically resolves carrier phase cycle count ambiguities via encoding of pseudo-random number codes on the power transmission signals. The system is capable of achieving phasing accuracies on the order of 3 mm down to 0.4 mm depending on whether the radio navigation beacons operate in the L or C bands.

## Work Accomplished During Grant Period

Two tasks were carried out under this grant. One was to analyze a concept for using RF ranging/timing techniques like those of the GPS system to do relative phasing control for the power transmission array of the Space Solar Power System (SSPS). The other was to develop and test a real-time GPS software receiver that could form the basis for the receiver hardware that is used in the SSPS phasing control system. A paper was completed on the software receiver concept and has been presented at a conference [1].

## Personnel

Two different researchers have been involved in this project: Hee Jung, a Ph.D. student, and Prof. Mark Psiaki. Both researchers were part of Cornell's Sibley School of Mechanical and Aerospace Engineering during all of this project. Hee Jung worked on developing some of the technology for the new software receiver concept. Prof. Psiaki collaborated with Hee Jung on her work, worked on other parts of the software receiver system, and did system-level studies of the SSPS transmission phasing control system.

## Significant Results

### SSPS Phasing Control

System Concept. The basic concept of the SSPS power transmission phasing control system is illustrated in Fig. 1. The purpose of the system is to phase the sinusoidal power signal that gets transmitted from each node so that the signals add constructively when they reach the power receiver at the Earth. This can be accomplished if all of the nodes have synchronized clocks and if all of the nodes know their relative range to the receiving antenna. Synchronization of the clocks and measurement of the relative ranges can be accomplished by using two GPS-like radio beacons, one located below the array at the power reception station on the Earth, and the other located above the array. Each node of the transmission array carries a pair of patch antennas for receiving the timing/ranging beacon signals and a radio receiver that tracks the two signals and measures their carrier phases. One of the nodes of the array is designated as the carrier phase leader. It transmits its beacon signal carrier phase measurements, referenced to its power signal transmission phase, to each of the other elements of the array. This transmission is performed using data bits which get encoded on beacon 1's signal. Each of the follower nodes in the array uses this transmitted phase information along with a cycle ambiguity calibration and its own carrier phase measurements in order to phase its own power transmission signal.

System Functional Interactions. Figure 2 depicts the interactions of the various components of the phasing control system in block-diagram format. The dotted and dash-dotted connections in the figure represent RF links, and the solid lines represent hard-wired links. The heavy dash-dotted lines towards the right-hand side of the figure represent the transmitted RF power signals from the array nodes. The 2 GPS-like RF beacons transmit low-power spread-spectrum signals in the L, S, or C frequency band. These signals are received by the leader node and by the follower nodes. The leader node uses these signals to generate phase measurements that it transmits back to the first RF beacon for later re-broadcast to the follower nodes. The leader node also uses its RF oscillator's output to phase its power transmission signal so that the transmission phase is linked to the phase measurements for the received beacon signals. The follower nodes use the beacon phases along with phase correction information from the leader node and from the ground power receiver to phase their power transmission signals. The ground power receiver periodically performs calibrations of the phase error relationship between the leader node and each follower node. This calibration information is sent to the first RF beacon for later re-broadcast to the follower nodes. The each node's power signal must have the capability of being modulated with a pseudo-random number (PRN) code for short periods of time so that the needed calibrations can occur. The first beacon must have the capability to have the phase correction and calibration data encoded on it for broadcast to the follower nodes.

One-Dimensional Analysis. The efficacy of this system can be understood by performing an analysis based on the geometry of Fig. 3. Each carrier phase measurement is a beat-phase that compares the received phase to a local oscillator's replica of the transmitted carrier frequency:

$$\phi_i^j(t) = \int_{t_0}^{t} [f_{LOi} - f_i^j(\tau)]d\tau \quad \text{for nodes } i = A,B \text{ \& for beacons } j = 1,2 \tag{1}$$

where $\phi_i^j$ is the carrier phase of the signal from beacon $j$ as measured by node receiver $i$, $f_{LOi}$, the local oscillator frequency of node receiver $i$, is the receiver's replica of the carrier signal's nominal transmitted frequency, and $f_i^j$ is the carrier frequency of the signal from beacon $j$ as measured by node receiver $i$. Suppose that the system takes the differences between the carrier phases from each beacon as received by the two different nodes and suppose that it multiplies these differences by the carrier wavelength. Then the following equations model these single-difference measurements:

$$\lambda_b[\phi_A^1(t) - \phi_B^1(t)] = -\frac{1}{\alpha_1}\ell \sin\theta(t) - \lambda_b N_{AB}^1 - c[\delta_{RA}(t) - \delta_{RB}(t)] \tag{2a}$$

$$\lambda_b[\phi_A^2(t)-\phi_B^2(t)] = +\frac{1}{\alpha_2}\ell \sin\theta(t) - \lambda_b N_{AB}^2 - c[\delta_{RA}(t)-\delta_{RB}(t)] \tag{2b}$$

where $\lambda_b$ is the carrier wavelength of each beacon's signal, $\ell$ and $\theta$ are defined in Fig. 3, $N_{AB}^j$ is the differential integer phase ambiguity between node receivers $A$ and $B$ for the signal from beacon $j$, $\delta_{Ri}$ is the local clock error for node receiver $i$ (the true time is the receiver clock time plus $\delta_{Ri}$), and $\alpha_1$ and $\alpha_2$ are factors near 1 that account for Doppler shifts and beacon frequency errors.

The two measurement equations (2a) and (2b) can be solved for the unknown relative range, $\ell\sin\theta$, and the unknown relative timing, $[\delta_{RA}-\delta_{RB}]$, if the integer ambiguities $N_{AB}^1$ and $N_{AB}^2$ are known. These solutions can be used to properly phase the transmitted power signal. The trick is to determine the integers $N_{AB}^1$ and $N_{AB}^2$. This need not actually be done. Instead, a combination of $N_{AB}^1$ and $N_{AB}^2$ can be determined that produces the proper phasing at the ground. This can be accomplished by a calibration procedure that involves a comparison of the received power signal carrier phases from nodes $A$ and $B$ at the ground reception station. This calibration can be determined intermittently because $N_{AB}^1$ and $N_{AB}^2$ are constants.

Before one can explain the calibration process, it is necessary to construct models of the transmitted and received power signals. The transmitted power signals from the two nodes are

$$y^A(t) = C^A \sin\{2\pi f_p t\} \tag{3a}$$

$$y^B(t) = C^B \sin\{2\pi f_p[t+\tilde{\delta}_{RA}(t)-\tilde{\delta}_{RB}(t)+\frac{\ell}{c}\sin\tilde{\theta}]\} \tag{3b}$$

where $C^i$ is the transmitted power signal amplitude from node $i$, $f_p$ is the nominal frequency of the transmitted power signal, and all quantities with the ~ overstrike are estimates. Notice how the follower node, node $B$, uses an estimate of $[\delta_{RA}-\delta_{RB}]$ and an estimate of $\ell\sin\theta$ in order to attempt to transmit its power signal with the correct carrier phase. The power signals that get received at the ground station are

$$y_1^A(t) = C^A G_{FS1A} \sin\{2\pi f_p[t-\frac{\rho_A^1}{c}+\delta_{RA}(t-\frac{\rho_A^1}{c})]\} \tag{4a}$$

$$\begin{aligned} y_1^B(t) = C^B G_{FS1B} \sin\{2\pi f_p[t-\frac{\rho_A^1}{c}-\frac{\ell}{c}\sin\theta+\delta_{RB}(t-\frac{\rho_A^1}{c}) \\ +\tilde{\delta}_{RA}(t-\frac{\rho_A^1}{c})-\tilde{\delta}_{RB}(t-\frac{\rho_A^1}{c})+\frac{\ell}{c}\sin\tilde{\theta}]\} \end{aligned} \tag{4b}$$

where $G_{FS1i}$ is the free-space signal attenuation factor from transmitter node $i$ to the ground power-reception antenna location. Note how the phases of the transmitted power signals differ from the phases of the received power signals because of two effects. One is the delay propagation time through free space, $(\rho_A^1)/c$. The other is the combined effect of the true node clock errors, $\delta_{RA}$ and $\delta_{RB}$, and the node-$A$/node-$B$ true articulation angle, $\theta$. These effects and the measured phase difference between the two received power signals can be used to calibrate the unknown integer ambiguities from the differential carrier phase solution so as to null out the difference in the received phases of the two power signals. Note, also, that eq. (4b) assumes that the receiver clock errors and their estimates do not vary significantly over the short time interval $\ell\sin\theta/c$.

The integer ambiguity calibration process starts by measuring the carrier phases of the two received signals. The phase measurements are

$$\psi_1^i(t) = \int_{t_0}^{t} [f_p - f_{R1}^i(\tau)] d\tau \quad \text{for transmitter nodes } i = A,B \tag{5}$$

where $\psi_1^i$ is the carrier phase of the power signal from transmitter node $i$ as measured by the ground power receiver, which is collocated with beacon 1, and where $f_{R1}^i$ is the carrier frequency of the power signal from transmitter node $i$ as measured by the ground power receiver. Note that the transmitter nodes will have to be transmitting PRN codes on their signals in order for the signals from nodes $A$ and $B$ to be distinguishable. The two power signal carrier phases are then used to form a single difference, which obeys the following equation:

$$\begin{aligned}
&[\psi_1^A(t+\frac{\rho_A^1}{c}) - \psi_1^B(t+\frac{\rho_A^1}{c})] + M_{AB}^1 \\
&\quad = f_p\{[\tilde{\delta}_{RA}(t) - \tilde{\delta}_{RB}(t)] - [\delta_{RA}(t) - \delta_{RB}(t)] + \frac{\ell}{c}[sin(\tilde{\theta}\{t\}) - sin(\theta\{t\})]\} \\
&\quad = \frac{\lambda_b}{\lambda_p}\left(\frac{1}{\alpha_1+\alpha_2}\right)[-\alpha_1(1+\alpha_2)\Delta N_{AB}^1 - \alpha_2(1-\alpha_1)\Delta N_{AB}^2]
\end{aligned} \tag{6}$$

where $M_{AB}^1$ is the differential integer carrier phase ambiguity between the power signals from transmitter nodes $A$ and $B$ as measured by the ground power receiver and $\lambda_p$ is the carrier wavelength of the power signal. The last line of eq. (6) has been derived using the beacons' carrier-phase models & the following definition of the integer ambiguity estimation errors: $\Delta N_{AB}^j = \tilde{N}_{AB}^j - N_{AB}^j$ for beacons $j$ = 1,2.

Given these differential carrier phase measurements for the power signals, the integer ambiguity resolution strategy is the following:

1. Periodically transmit a PRN code on the node-$A$ power signal and a different PRN code on follower node $B$'s power signal.
2. Measure the carrier phase difference $\psi_1^A$-$\psi_1^B$.
3. Choose (non-unique) integers $M_{AB}^1$, $\Delta N_{AB}^1$, and $\Delta N_{AB}^2$ to achieve an acceptably small error in the power signal phase measurement model of eq. (6).
4. Use the $\Delta N_{AB}^1$ and $\Delta N_{AB}^2$ values from Step 3 to update the estimates $\tilde{N}_{AB}^1$ and $\tilde{N}_{AB}^2$ that get used in eqs. (2a) and (2b) to determine the estimates of $\ell sin\theta$ and $[\delta_{RA}$-$\delta_{RB}]$ that are used in eq. (3b) to adjust node $B$'s transmitted power signal phase.

The main point of eq. (6) is that the non-integer part of the transmitted power signals' phase difference, $\psi_1^A$-$\psi_1^B$, is a function of $\Delta N_{AB}^1$ and $\Delta N_{AB}^2$. Step 4 of the ambiguity resolution process makes corrections which ensure that the fractional part of the received power signal phase error, $\psi_1^A$-$\psi_1^B$, will become small. This process is not guaranteed to correctly resolve the absolute integer ambiguities $N_{AB}^1$, $N_{AB}^2$, and $M_{AB}^1$, but exact resolution is unimportant as long as the fractional phase error in $\psi_1^A$-$\psi_1^B$ is small because it is the fractional part that governs whether the two nodes' signals add constructively or destructively at the ground-based power reception antenna.

System Performance. The system's phase measurements can be made to an accuracy of 1% of a carrier wave length. For L-band beacon signals this translates into a precision of 3 mm or better for relative positioning and 10 psec or better for relative timing. Higher precision can be achieved with S- or C-band beacons. Partial integer ambiguity resolution will allow this same level of precision to be achieved for the power signals' phasing control. This close correlation between the system's raw measurement precision and its phasing control is caused by the low geometrical dilution of precision that results from using beacon signals that are located both above and below the transmitter array.

System Component Functions. The system's components must have the following functional capabilities:

GPS-Like RF Beacons:

They must be at known locations. They must transmit L, S, or C-band carrier signals with a unique PRN code for each beacon. The PRN code chipping rate should be on the order or 10 MHz and the transmitting antenna should be circularly polarized. Both of these features tend in minimize the multi-path interference effects of reflected signals. In addition to the PRN code, at least one of the beacon signals must be able to transmit received beacon carrier phase measurement data from the phase leader node. One of the beacons must also transmit the received power signals' single-differenced carrier phases for each follower node. These transmissions occur periodically, whenever a re-calibration is performed.

Phase Leader Receiver/Transmitter:

This receiver must measure the received phase of each beacon signal and must synchronize these measurements to the phase of its transmitted power signal. These carrier phase measurements must be transmitted to an RF beacon for re-broadcast to the follower nodes. This node must have the ability to intermittently mix a PRN spreading code with its transmitted power signal for use in resolution of carrier phase ambiguities at the follower nodes.

Phase Follower Receiver/Transmitter:

Each of these receivers must measure the received phase from each beacon signal. It must use these carrier phases in a single-difference calculation that allows it to solve a one-dimensional navigation problem for its relative range and timing as measured with respect to the leader node. This calculation uses leader-node carrier phase data and ambiguity calibration data that it decodes from beacon signals. This sub-system uses its navigation/timing solution to control the phase of its transmitted power signal in such a way that the phase will add constructively to the leader node's power signal phase at the power reception antenna. These nodes also must have the ability to intermittently mix a PRN spreading code with their transmitted power signals for use in resolution of carrier phase ambiguities at the follower nodes. Note that each node must use a unique PRN spreading code in this calibration process.

Power Receiver:

This sub-system must measure integer ambiguity calibration data by correlating its received signals with internal replicas of the transmitter nodes' PRN codes. Calibration happens only intermittently because the calibrations are integer numbers of cycles, which, in theory, do not change with time. In practice, they change only when carrier tracking cycle slips occur. This sub-system must be able to output its integer ambiguity calibration data to the collocated first RF beacon.

Structure:

The structure must provide in-plane stiffness so that the in-plane relative locations of the nodes are known. This information is needed in order to do triangulation of signals. Out-of-plane bending is allowed because the beacon-based measurements detect this bending and compensate for it. The in-plane stiffness must maintain mm-level in-plane relative positioning accuracy if the second RF beacon is close to the array. As the 2nd beacon gets located further and further above the constellation, it becomes permissible to allow larger in-plane relative articulation uncertainties between the array elements because the performance of the phasing system becomes less sensitive to these uncertainties.

Open Issues Regarding System Design and Performance. There are a number of open issues that still need to be investigated before an actual system gets designed. Open questions about the beacons include: Where is the best place to put overhead beacon #2? Might it be wise to orbit a formation of unattached beacon's around the array so that there would be a sequence of beacons #2 each of which would be far away from the array? How accurately does beacon #2's location need to be known, and how stable do the beacons' local oscillators need to be? How strong do the beacon signals need to be? What is the optimal PRN code chipping rate and code length to use for the beacon signals? Additional general open questions about the system are: What are the effects of an off-nominal array attitude and an off-nominal overhead beacon location? Does there need to be additional compensation for the second-order effects of array out-of-plane bending, and if so, how should this be carried out? What is the most cost effective hardware implementation? Can the system be integrated with an RF-based attitude determination system for each transmitter node? Given that the finite thickness of a node will cause a radial displacement between the antenna that receives the beacon 1 signal and the antenna for the beacon 2 signal, will this displacement between the two antennas cause any problems for the carrier phase control system?

**Software Radio Receiver**

A GPS receiver has been developed that runs 12 tracking channels in real time using a software correlator. This work is part of an effort to develop a flexible receiver that can use new GPS-like signals without the need for new correlator hardware. The receiver consists of an RF front end, a system of shift registers, a digital data acquisition card, and software that runs on a 1.73 GHz PC. The PC performs base-band mixing and PRN code correlations in a manner that directly simulates a hardware digital correlator. It also performs the usual signal tracking and navigation functions, under the control of a real-time Linux operating system. The contributions of this work are a set of special high-speed algorithms for doing the correlations in software. They make use of bit-wise parallelism to process 32 samples simultaneously. This system has been tested using a stationary antenna. When operating with 12 channels the receiver uses less than 35% of the capacity of the processor and navigates to an accuracy of 10 meters [1].

In addition, a new RF front-end has been developed and tested in conjunction with the real-time software receiver. It is based on direct RF sampling. This technique passes the raw RF signal through a band-pass filter and then samples it at a carefully selected frequency that is slightly greater than four times the chipping rate of the received signal's PRN code. This results in RF down conversion through intentional aliasing. The Cornell work on this system represents the first continuous implementation in a radio navigation system of the ideas that were first tested in Ref. 2.

A system based on direct RF down-conversion and real-time software radio is attractive for use in the SSPS transmitter phasing control system. It can be used to implement the radio navigation receivers that must operate on each transmission array node. This technology will allow these receivers to be designed and built mostly from commercial off-the-shelf technology (COTS) instead of from specially designed RF hardware. Most of the receiver could be implemented using a general-purpose DSP chip. Such systems are very flexible. Changes can be made merely by reprogramming software in the DSP, which will allow the system to be adapted should flight experience show that changes are needed.

**Inventions**

This work has resulted in one invention for which a provisional patent application has been filed. The patent application is for a set of base-band mixing and PRN code correlation algorithms that are needed to implement the real-time software GPS receiver. They use bit-wise parallelism in order to speed up the needed calculations by a factor of 2 or 3. The inventors are B.M. Ledvina, M.L. Psiaki, S.P. Powell, and P.M. Kintner, Jr. The title of the invention is: "Real-Time Software Receiver for GPS and Wireless Applications." A provisional patent application was filed on Jan. 10, 2003. The patent is pending. It's Cornell reference number is CRF D-3164.

## Summary

This grant has supported efforts to develop a system concept and components for a phasing control system for the power transmission array of the Space Solar Power System. The system concept consists of a pair of radio-navigation beacons and a set of radio navigation receivers, one for each node of the transmission array. One beacon is located overhead of the array, and the other is located at the ground reception station. Carrier phase differential relative ranging/timing techniques allow these components to control the power signal transmission phasing so that all of the transmitters' signals arrive in phase at the ground-based power reception antenna. Ambiguities about the integer numbers of carrier cycles are satisfactorily resolved through the periodic use of a calibration process which employs pseudo-random number codes that get mixed with the power transmission signals. Hardware developments have produced real-time software radio receiver technology that can be used to implement the needed radio navigation receivers.

## References


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2. Akos, D.M., and Tsui, J.B.Y., "Design and Implementation of a Direct Digitization GPS Receiver Front End," *IEEE Trans. on Microwave Theory and Techniques*, 44(12), 1996, pp. 2334-2339.


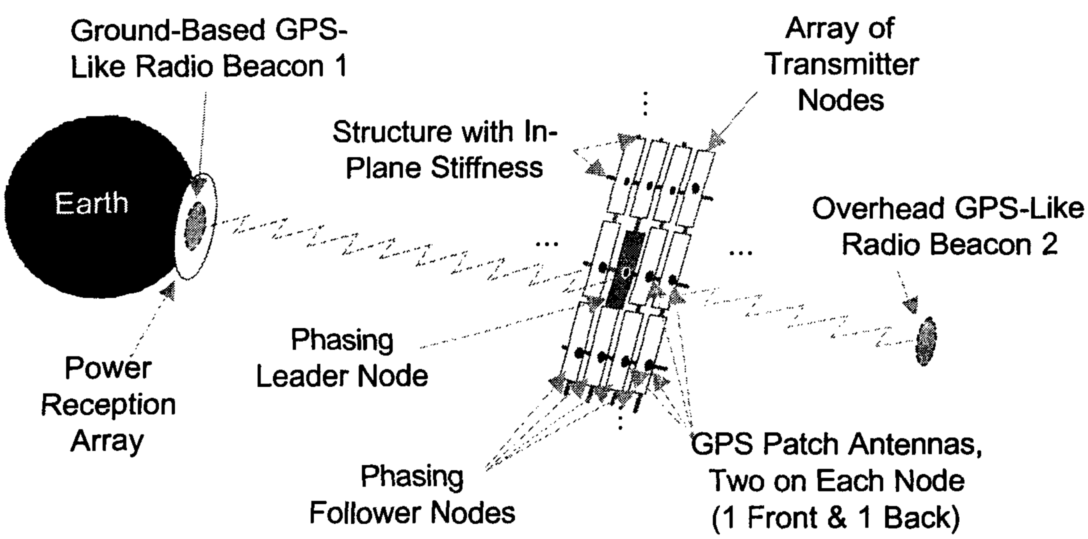


*Fig. 1. Transmitter node phasing sub-system for the SSPS (not to scale).*

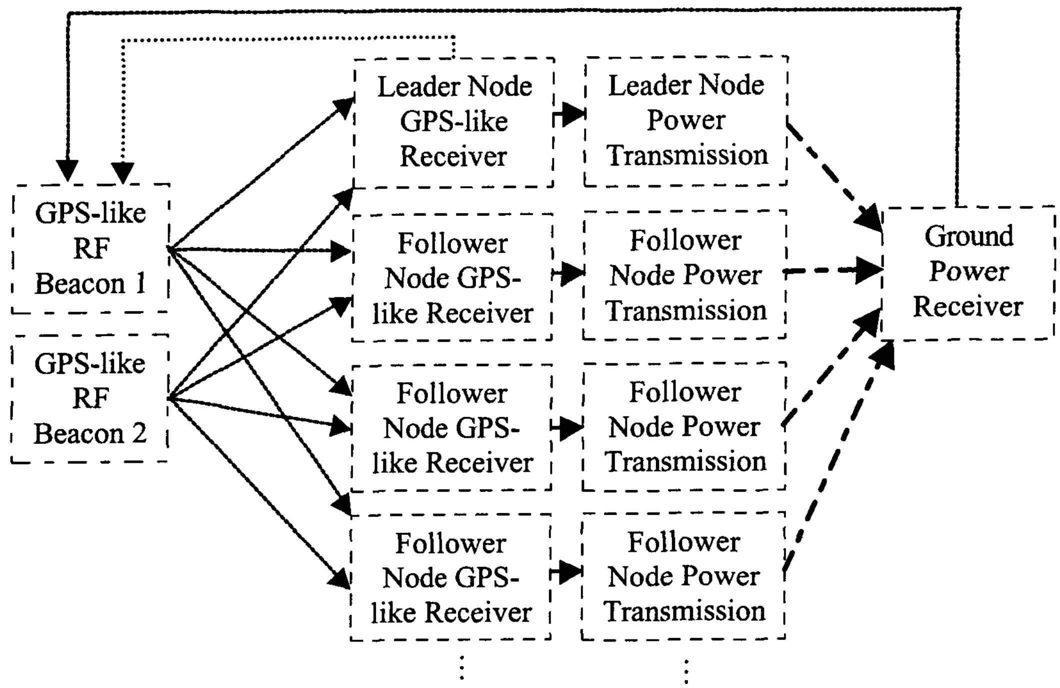


*Fig. 2. System block diagram.*

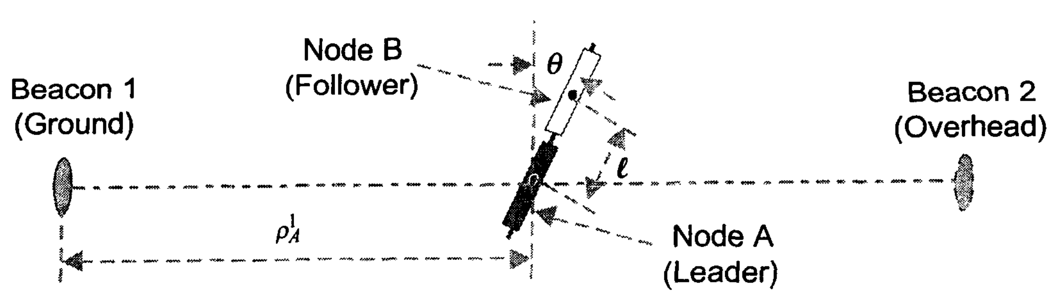


*Fig. 3. One-dimensional carrier-phase-based relative positioning and timing.*